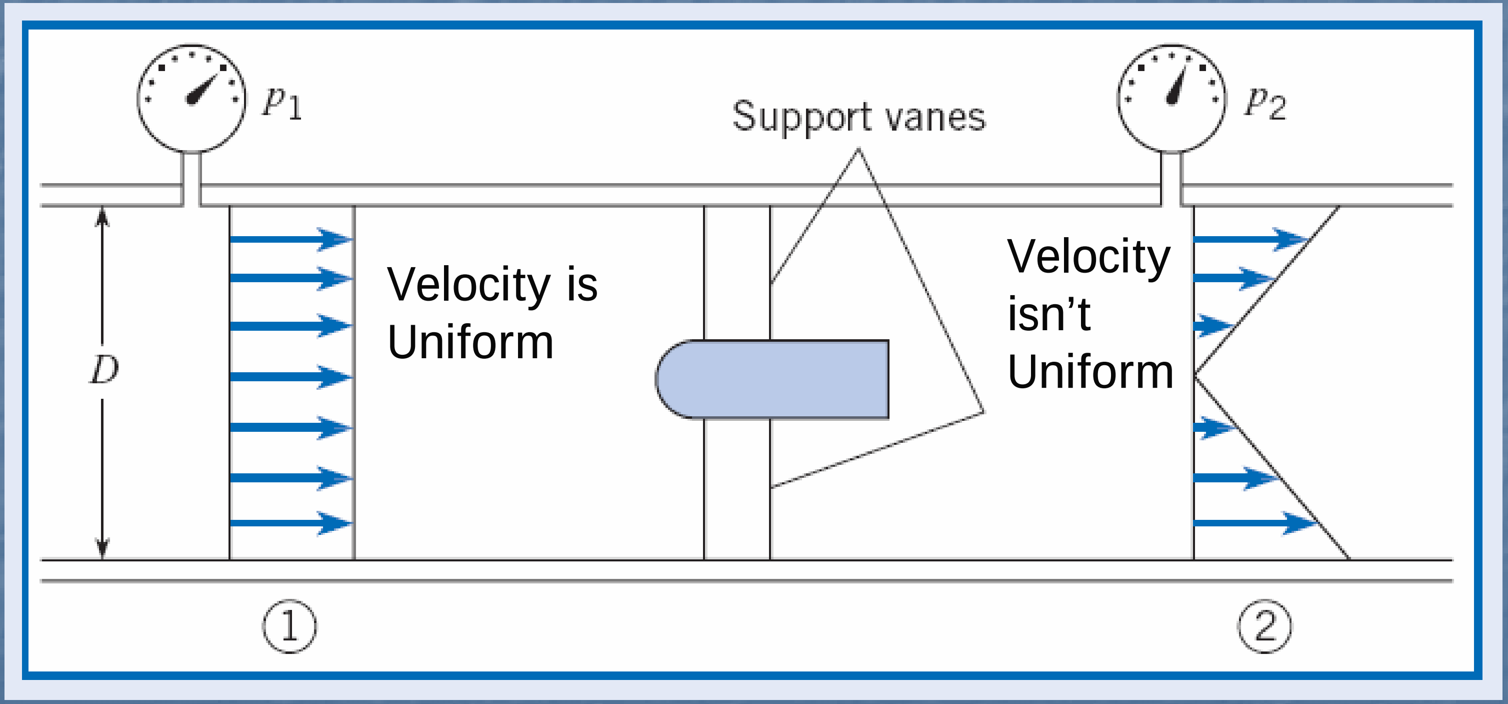
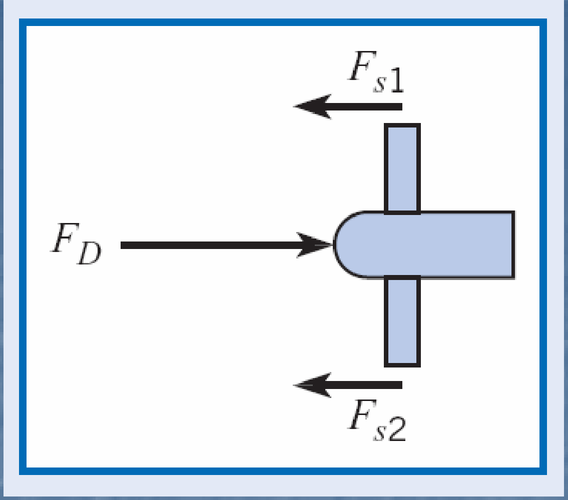
Non-uniform Velocity Distribution

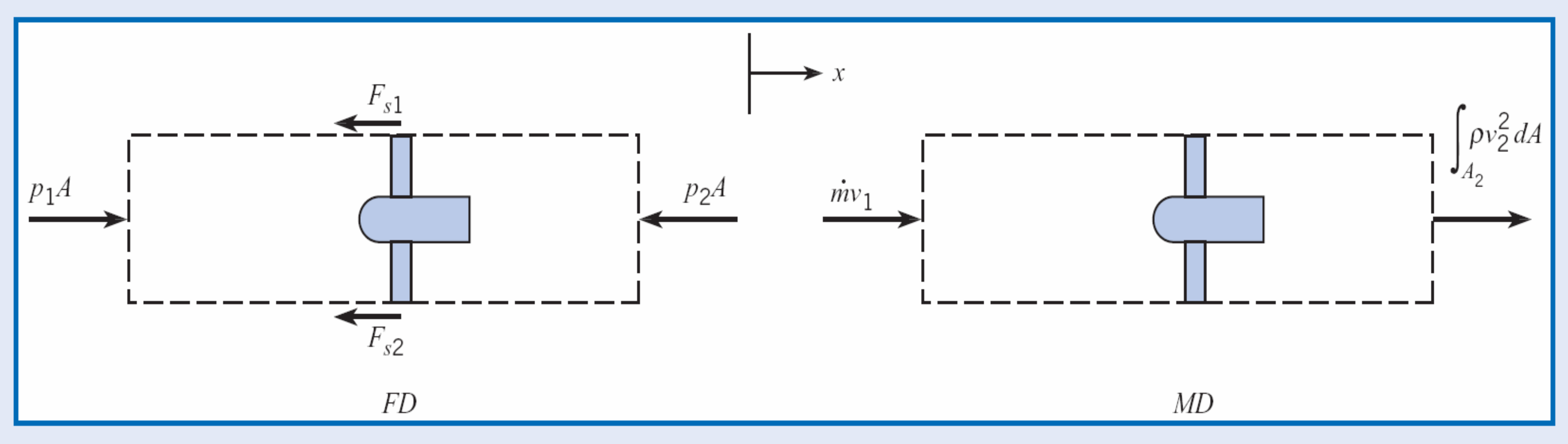
Example 6.8

The drag force of a bullet-shaped device may be measured using a wind tunnel. The tunnel is round with a diameter of 1 m, the pressure at section 1 is 1.5 kPa gage, the pressure at section 2 is 1.0 kPa gage, and air density is 1.0 kg/m^3. At the inlet, the velocity is uniform with a magnitude of 30 m/s. At the exit, the velocity varies linearly as shown in the sketch. Determine the drag force on the device and support vanes. Neglect viscous resistance at the wall, and assume pressure is uniform across sections 1 and 2.



Find: The drag force on the device and the vanes?





Given: $v_1 = 30\, m/s$, $p_1 = 1.5\, kPa\ gauge$, $p_2 = 1.0\, kPa\ gauge$, $\rho_{air} = 1.0\, kg/m^3$, $D = 1m$

This problem involves forces in the (x) direction

$$\frac{d}{dt}\int_{cv} v_Z \rho dQ = 0$$ (Flow is steady)

$$F_D = F_{S1} + F_{S2}$$

The drag force is balanced by the forces on the support vanes

$$\sum F_x = p_1 A_1 - p_2 A_2 - (F_{S1} + F_{S2})$$

From the force diagram, $\sum F_x = p_1 A_1 - p_2 A_2 - F_D$

From the Momentum Diagram,

$$\sum F_x = \frac{d}{dt}\int_{cv} v\rho dQ + \int_{cs} v\rho V \bullet dA = \int_{CS} v\rho v \bullet dA$$

$$\int_{cs} v\rho V \bullet dA = \int_{A1} v_1 \rho v_1 dA = -\dot{m} v_1$$

$$\int_{cs} v\rho V \bullet dA = \int_{A2} v_2 \rho v_2 dA = \int_{A2} \rho v^2 dA$$

The velocity at the exit is variable and changes linearly with radius (r), so the function for v_2

$$v_2 = v_{\max}\left(\frac{r}{r_0}\right) \qquad (r_0 = \frac{D}{2})$$

$$\int_{A2} v_2 \rho v_2 dA = \int_{A2} \rho v_2{}^2 dA = \int_0^{D/2} \rho \left[v_{\max}\left(\frac{r}{r_0}\right)\right] 2\pi r\, dr$$

$$\sum F_x = \sum_{CS} (\dot{m}v)_{out} - \sum_{CS} (\dot{m}v)_{in}) = \int_0^{D/2} \rho \left[v_{\max}\left(\frac{r}{r_0}\right)\right] 2\pi r dr - \dot{m}v_1$$

$$-F_D + p_1 A_1 - p_2 A_2 = \int_0^{D/2} \rho \left[v_{\max}\left(\frac{r}{r_0}\right)\right] 2\pi r dr - \dot{m}v_1$$

$$\sum F_x = \sum_{CS} (\dot{m}v)_{out} - \sum_{CS} (\dot{m}v)_{in}) = \int_0^{D/2} \rho \left[v_{\max} \left(\frac{r}{r_0} \right) \right] 2\pi r dr - \dot{m}v_1$$

$$-F_D + p_1 A_1 - p_2 A_2 = \int_0^{D/2} \rho \left[v_{\max} \left(\frac{r}{r_0} \right) \right] 2\pi r dr - \dot{m}v_1$$

$$A_1 = A_2 = A = \frac{\pi D^2}{4}$$

$$(v_{\max}) = 45\, m/s$$

$$-F_D = (p_2 - p_1)A + \int_0^{D/2} \rho \left[v_{\max} \left(\frac{r}{r_0} \right) \right] 2\pi r dr - (\rho A v_1{}^2)$$

The term $\int_0^{D/2} \rho \left[v_{\max} \left(\frac{r}{r_0} \right) \right] 2\pi r dr = A_1 v_1$

$$(\pi \times 0.5^2 \text{ m}^2)(30 \text{ m/s}) = \int_0^{0.5} v_{\max}(r/0.5) 2\pi r\, dr$$

which can be solved to show that $v_{max} = 45$ m/s. Evaluating the exit momentum flow gives

$$\int_{A_2} \rho v_2^2 \, dA = \rho \int_0^{0.5} v_{max}^2 (r/0.5)^2 2\pi r \, dr$$

$$= (1.0 \text{ kg/m}^3)(45^2 \text{ m}^2/\text{s}^2)(0.5^2 \text{ m}^2)(\pi/2)$$

$$= 795.2 \text{ N}$$

The drag force is

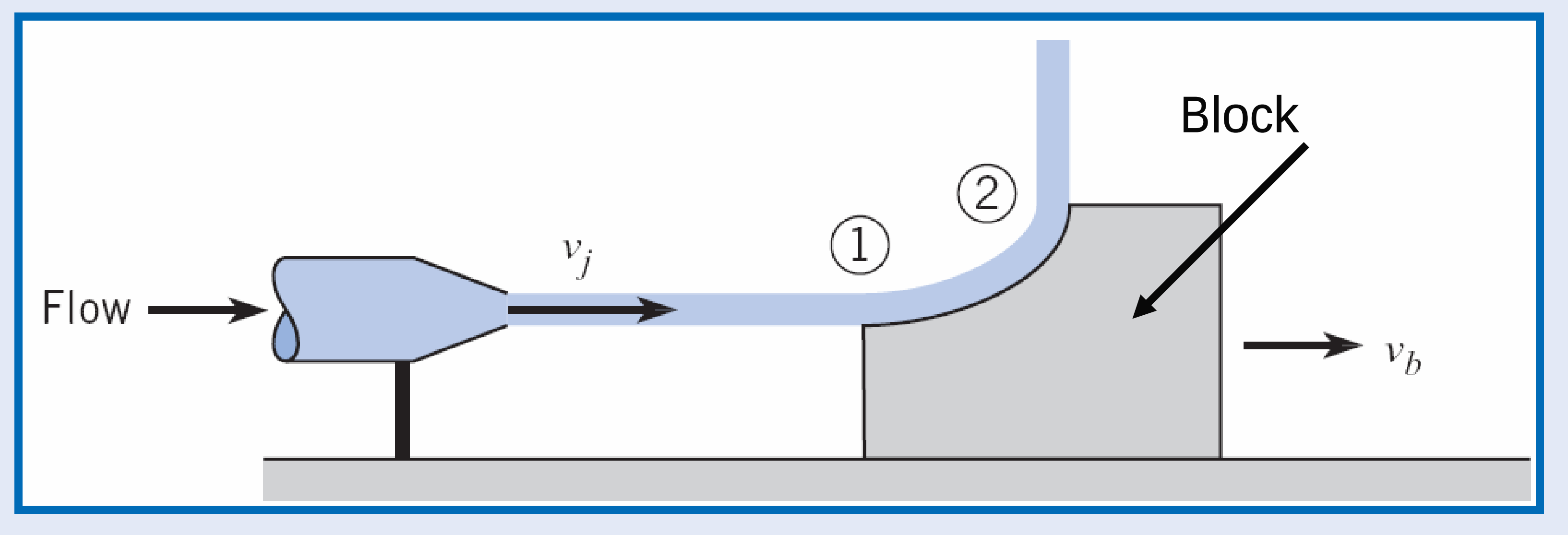
$$F_D = A_1(p_1 - p_2) + \dot{m} v_1 - \int_{A_2} \rho v_2^2 \, dA$$

$$= 392.7 \text{ N} + 706.8 \text{ N} - 795.2 \text{ N}$$

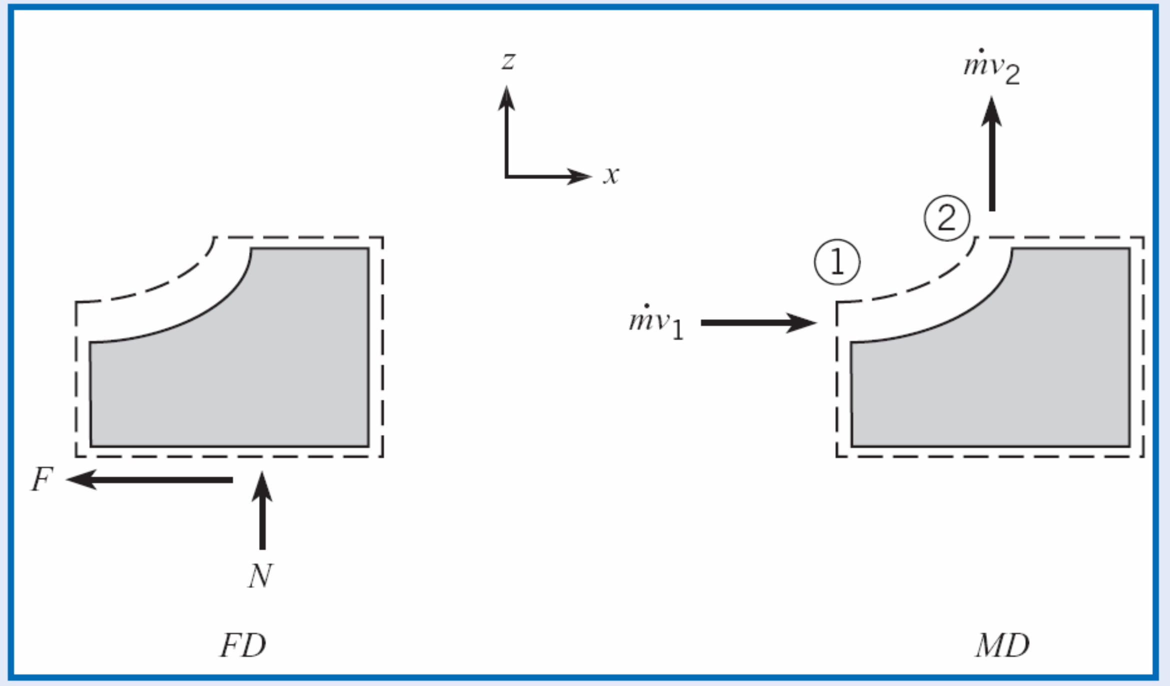
$$= 304 \text{ N}$$

Example (6.9)

A stationary nozzle produces a jet with a speed v_j and an area A_j. The jet strikes a moving block and is deflected 90° relative to the block. The block is sliding with a constant speed v_b on a rough surface. Find the frictional force F acting on the block.



Find: The frictional force (F) acting on the block?



This problem involves forces in the (x,z) direction

The momentum accumulation = $\frac{d}{dt}\int_{cv} v_z \rho dQ = 0$ (Flow is steady)

From the force diagram,

$$\sum F_x = -F$$

$$\sum F_z = N - W_{block}$$

From the momentum diagram,

$$\sum F_x = \sum_{CS} (\dot{m}v)_{out} - \sum_{CS} (\dot{m}v)_{in}) = 0 - (\dot{m}v_1) = -\dot{m}v_1$$

$$\sum F_x = -F = -\dot{m}v_1$$

$$\sum F_Z = \sum_{CS} (\dot{m}v)_{outZ} - \sum_{CS} (\dot{m}v)_{inZ} = \dot{m}v_2 - 0 = \dot{m}V_2$$

$$\sum F_Z = N - W_{block} = \dot{m}v_2$$

The mass flow rate is calculated using the velocity relative to the control surface $(v_j - v_b)$, and so

$$\dot{m} = \rho A_j (v_j - v_b)$$

Notice that $\dot{m} \longrightarrow 0$ as $v_b \longrightarrow v_j$, which should be the case because $\dot{m}$ is the rate at which mass is crossing the control surface.

The speed v_1 is relative to the moving reference frame, and so

$$v_1 = v_j - v_b$$

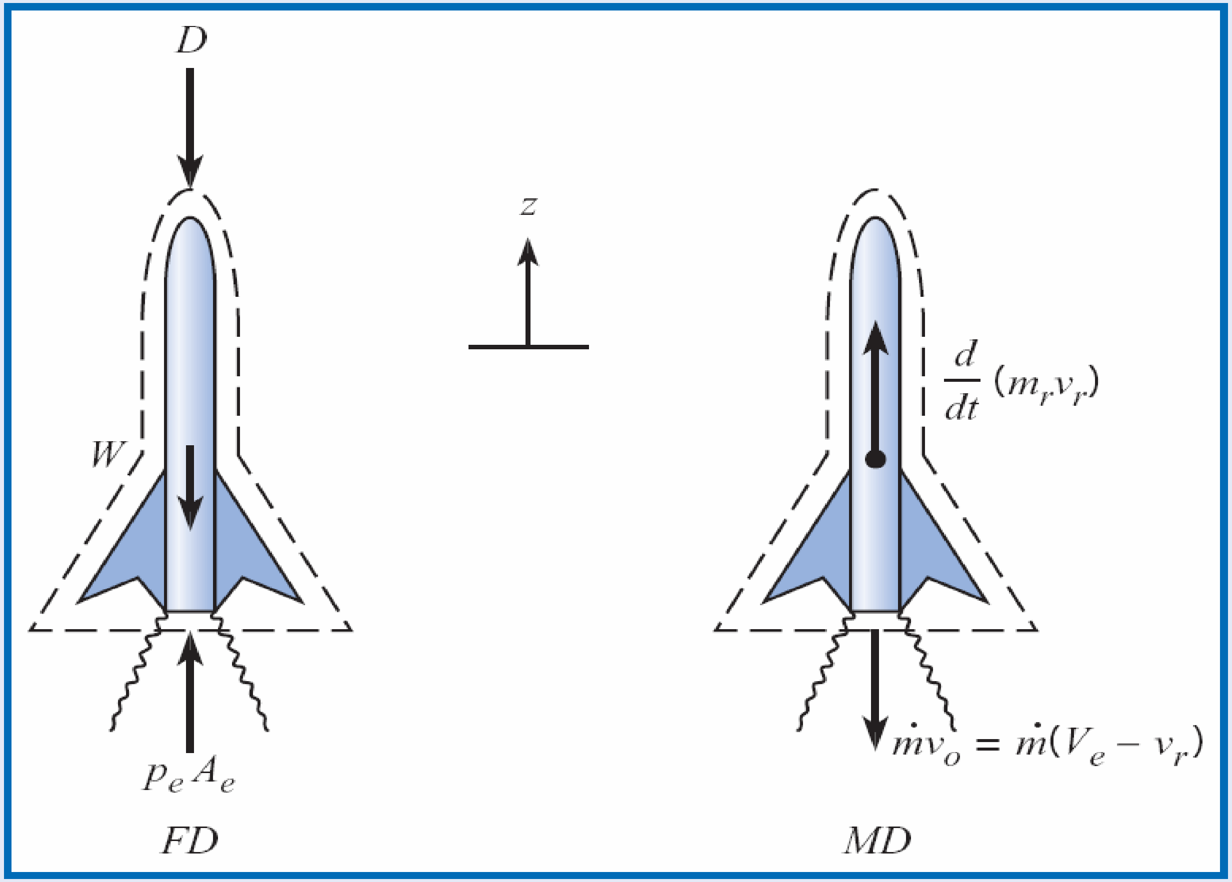
Combining terms results in

$$F = \dot{m} v_1 = \rho A_j (v_j - v_b)^2$$

Example (6.10)

Consider a rocket of mass m_r traveling at a speed v_r as measured from the ground. Exhaust gases leave the engine nozzle (area A_e) at a speed V_e relative to the nozzle of the rocket, and with a pressure that is higher than local atmospheric pressure by an amount p_e. The aerodynamic drag force on the rocket is D. Derive an equation for the acceleration of the rocket.

Find the accelerating $\frac{dv_r}{dt}$ **of the rocket?**



ANALYSIS

- We select a control volume moving with the rocket.
- We select a reference frame fixed to the launch pad.

This problem involves forces in the (z) direction

From the force diagram, $\sum F_z = p_e A_e - D - W$

From the momentum diagram,

$$\sum F_Z = \frac{d}{dt}\int_{cv} v_Z \rho dQ + \sum_{CS} (\dot{m}v)_{outZ} - \sum_{CS} (\dot{m}v)_{inZ}$$

The momentum accumulation = $\frac{d}{dt}\int_{cv} v_Z \rho dQ \neq 0$

$$\frac{d}{dt}\int_{cv} v_Z \rho dQ = \frac{d}{dt}\left(v_r \int_{cv} \rho \, dQ \right) = \frac{d}{dt}(m_r v_r)$$

In this development, we assumed that all parts of the rocket were traveling at the same speed v_r. The outward momentum flow is $\dot{m}v_o$, where v_o is relative to the stationary reference frame:

$$\sum_{CS} (\dot{m}v)_{out} = -(\dot{m}v_0) = -\dot{m}(v_e - v_r)$$

$$\sum_{CS} (\dot{m}v)_{in} = -(\dot{m}v_{in}) = 0$$

$$\sum F_Z = \frac{d}{dt}(m_r v_r) - \dot{m}(v_e - v_r)$$

$$p_e A_e - W - D = \frac{d}{dt}(m_r v_r) - \dot{m}(v_e - v_r)$$

$$\frac{d}{dt}(m_r v_r) = m_r \frac{dv_r}{dt} + v_r \frac{dm_r}{dt} = m_r \frac{dv_r}{dt} - \dot{m}v_r$$

$$m_r \frac{dV_r}{dt} = (\dot{m}V_e + p_e A_e) - D - W$$

$$m_r \frac{dV_r}{dt} = (\dot{m} V_e + p_e A_e) - D - W$$

$$\dot{m} = \rho A_e v_e$$

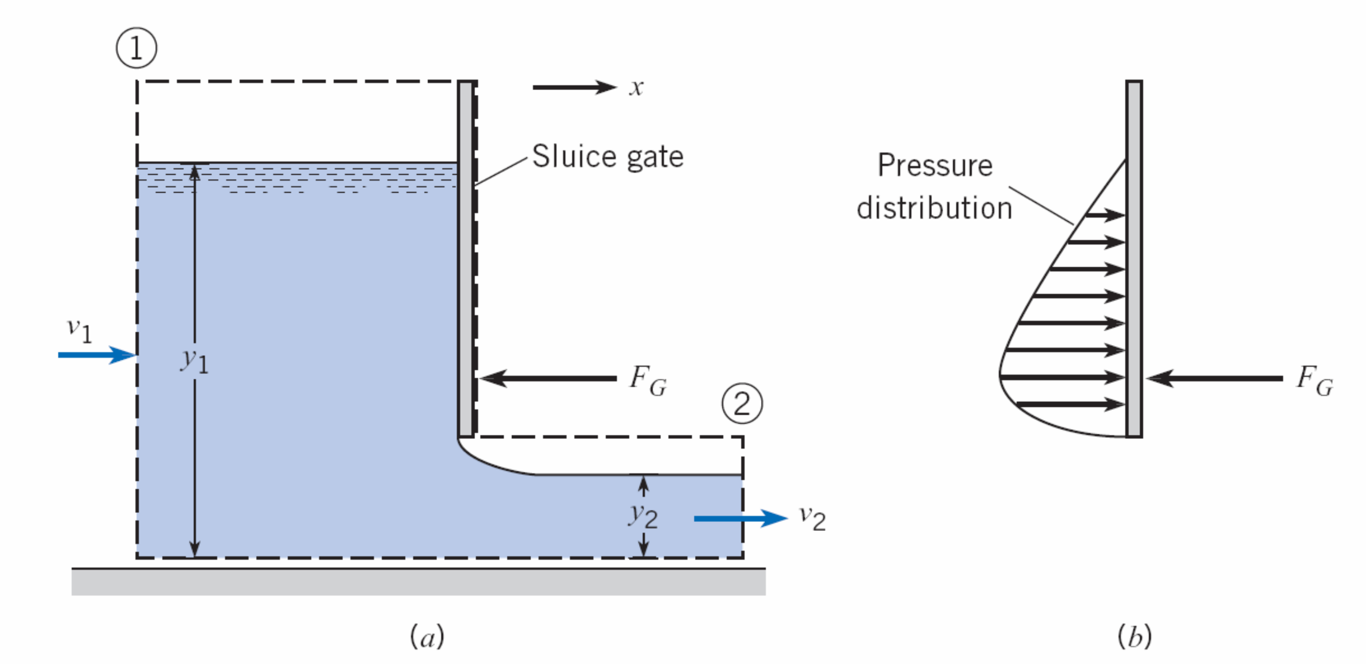
$$m_r \frac{dV_r}{dt} = (\rho A_e V_e) V_e + p_e A_e) - D - W$$

$$m_r \frac{d}{dt} v_r = (\rho A_e v_e{}^2 + p_e A_e) - W - D$$

The acceleration of the rocket $= \frac{dv_r}{dt} = \frac{(\rho A_e v_e{}^2 + p_e A_e) - D - W}{m_r}$

The term $(\rho A_e v_e{}^2 + p_e A_e)$ is known **as the thrust of the rocket motor (T)**

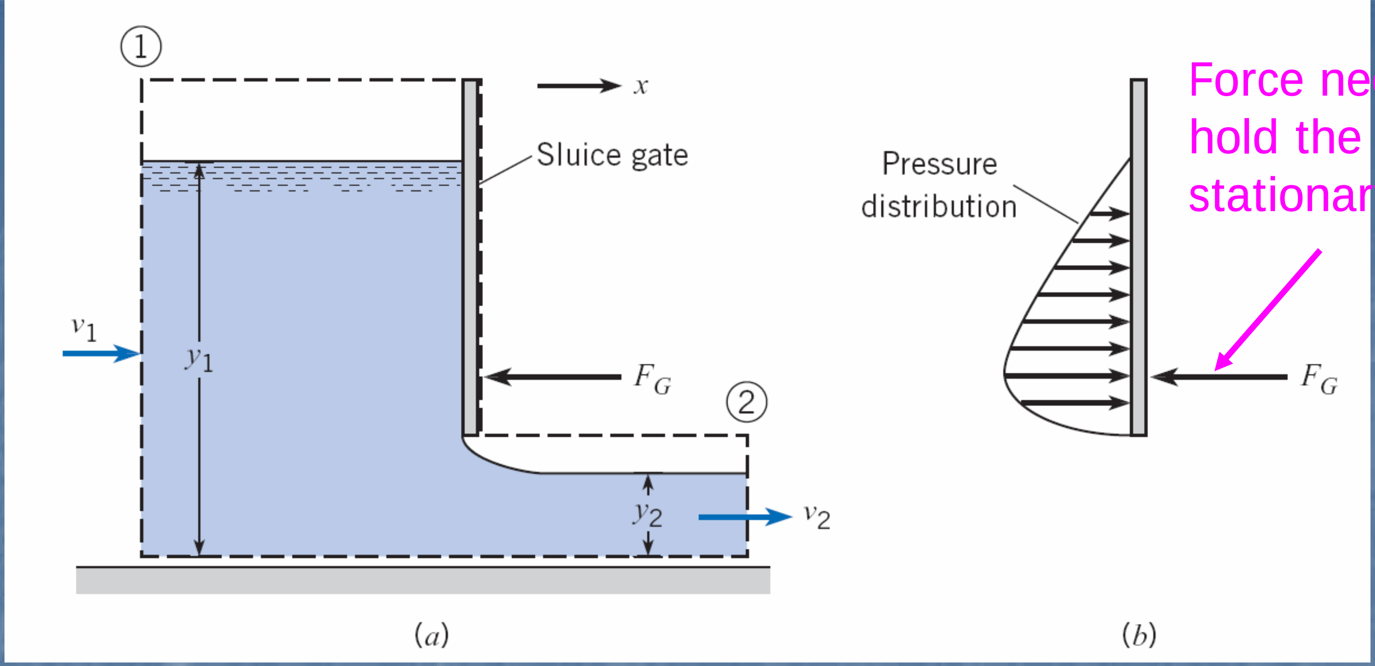
Force on a Rectangular Sluice Gate



ANALYSIS;

- Velocities near the upper part of the gate are quite low, therefore, the pressure distribution is approximately hydrostatic.
- Velocities near the lower part of the gate are quite large.

- The momentum accumulation = $\frac{d}{dt}\int_{cv} v_z \rho dQ = 0$



Force needed to hold the gate stationary

From the force diagram,

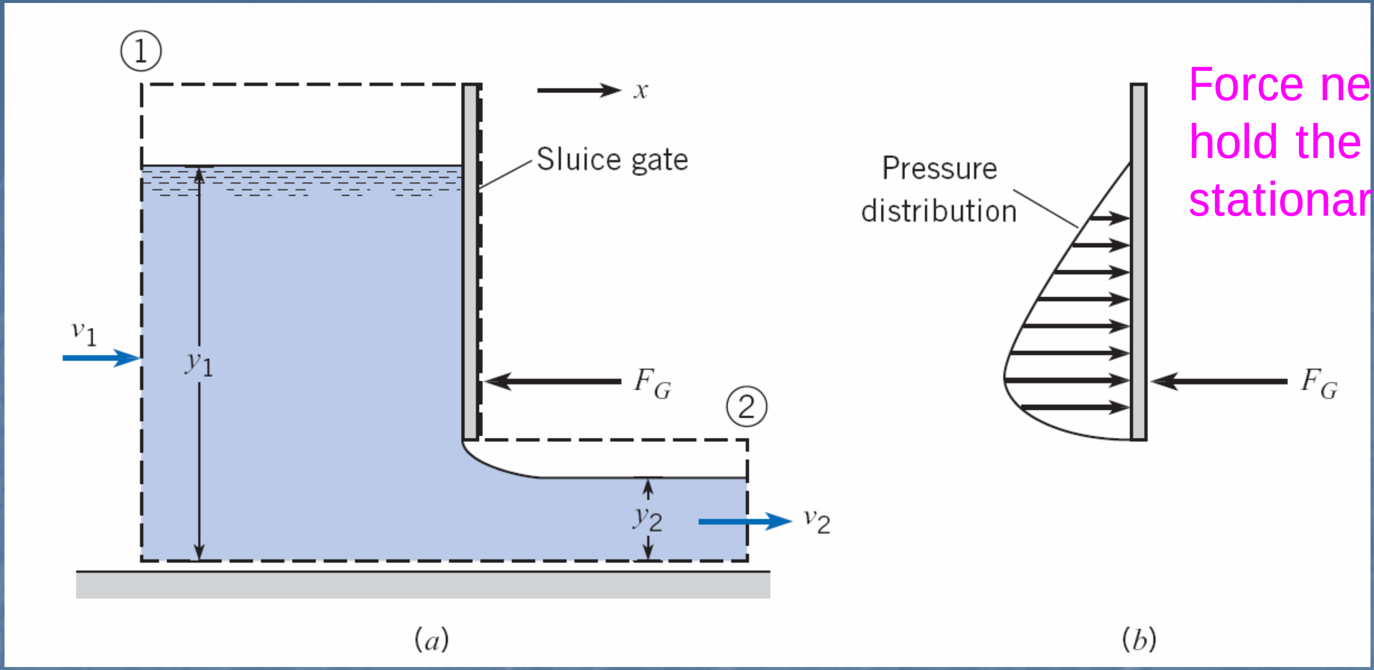
$$\sum F_x = \bar{p}A_1 - \bar{p}A_2 - F_G - F_{shear}$$

$$But \quad -F_G = -\int_{A_W} p\,dA, \quad F \approx 0$$

From the momentum diagram,

$$\sum F_x = \sum_{CS} (\dot{m}v)_{out} - \sum_{CS} (\dot{m}v)_{in}) = (\dot{m}v_2) - (\dot{m}v_1)$$

$$\sum F_x = \bar{p}A_1 - \bar{p}A_2 - F_G - F_{viscous} = \dot{m}(v_2 - v_1) = \rho Q(v_2 - v_1)$$



Force needed to hold the gate stationary

$$\bar{p}_1 A_1 = \gamma\left(\frac{y_1}{2}\right)\times(y_1 b)$$

$b = width\ of\ the\ gate$

$$\bar{p}_2 A_2 = \gamma\left(\frac{y_2}{2}\right)\times(y_2 b)$$

$$F_{viscous} = 0$$

$$F_G = \left(\frac{\gamma b}{2}\right)\left(y_2^{\ 2} - y_1^{\ 2}\right) + \rho Q\,(v_2 - v_1)$$

END OF LECTURE (5)